Bounds for the Solar Scatter Angle Observed from Earth Orbit

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Theme

THE geometry of solar reflections received by a sensor orbiting the Earth is analyzed. An earlier lower bound for the scatter angle α is sharpened considerably and allowable regions in a three-dimensional parameter space are determined.

Contents

Solar reflections from clouds are a consideration of some importance in the design and analysis of a configuration of Earth-orbiting satellites. Additionally, delicate sensors must usually be restricted from pointing too closely to the Sun. Consequently, analyses of coverage[1,2] must be augmented by a study of undesirable solar reflections. Since the scatter angle α, which is defined as the angle between the sensor line-of-sight and the Sun, is the principal measure of intensity of these reflections, bounding α from below for all time is a consideration in choosing a satellite network.[3]

Previous work[4] considered both the direct problem of obtaining various sensor-Sun angles for a prescribed sensor scanning motion and the indirect problem of obtaining, at a given time, loci of constant scatter angle α. Also, the effect on α of a variation in orbit altitude was examined and a rather crude lower bound for α was obtained. In particular, this lower bound was only satisfactory for rather high altitude orbits (for example, $h > R_\oplus$).

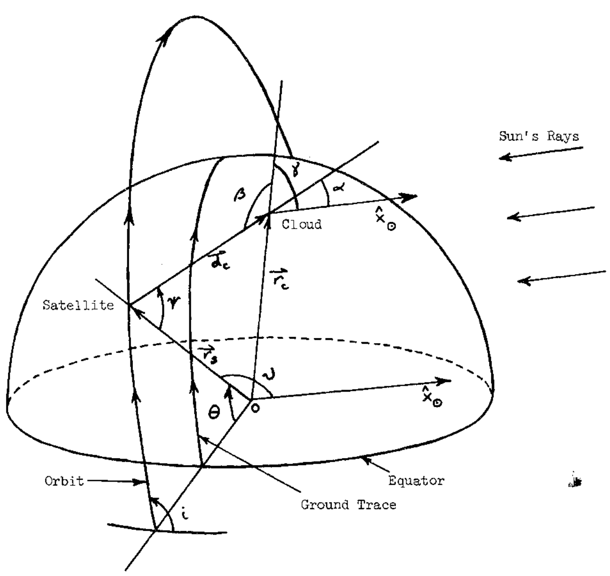

Fig. 1 Satellite-Cloud-Sun geometry.

In this analysis, the following assumptions were made: a) the satellite is to be in a circular or elliptic orbit about the Earth; b) the satellite field-of-view extends from the nadir to the horizon; c) the eccentricity of the Earth's orbit about the Sun ($e = 0.016726$) is neglected; and d) a spherical Earth is assumed to be totally covered by a cloud layer at 30,000 ft alt. Subject to these assumptions, the problem of low scatter angle reflections is analyzed and an exact lower bound for α, α_L, is determined. Allowable regions in a three-dimensional parameter space appropriate to cos α are delineated and the limiting values of α, which occur on the boundaries of this space, are determined. Specific results for a low altitude (400 nm) satellite are illustrated elsewhere.[5]

As shown in Fig. 1, the solar-sensor geometry is described in terms of the vectors $\boldsymbol{r}_s$, $\boldsymbol{r}_c$, $\boldsymbol{d}_c = \boldsymbol{r}_c - \boldsymbol{r}_s$, and $\hat{\boldsymbol{x}}_\odot$ (a unit vector to the Sun). The following angles are of principal interest:

$$\alpha = \text{scatter angle obtained from } \boldsymbol{d}_c \cdot \hat{\boldsymbol{x}}_\odot = d_c \cos\alpha$$

$$\gamma = \text{entrance angle obtained from } \boldsymbol{r}_c \cdot \hat{\boldsymbol{x}}_\odot = r_c \cos\gamma \quad (1)$$

$$\nu = \text{Sun angle obtained from } \boldsymbol{r}_s \cdot \hat{\boldsymbol{x}}_\odot = r_s \cos_\nu$$

Other angles are also needed. Consider the triangle with vertices at satellite, cloud, and center of the Earth shown in Fig. 1. The exit angle β, elevation angle ψ, and angular separation, $\beta - \psi \triangleq \delta$ between satellite and cloud are shown. The law of sines gives

$$\sin\beta / r_s = \sin\psi / r_c = \sin\delta / d_c \quad (2)$$

which implies $\beta > \psi$. Also, using the Keplerian orbit equation

$$r_s = a(1-\epsilon^2)/[1+\epsilon\cos(\theta-\omega)] \quad (3)$$

we get

$$\sin\psi \leq r_c / a(1+\epsilon) \quad (4)$$

as necessary restrictions on the elevation angle ψ for a given orbit (a, ϵ).

Defining the dimensionless ratio

$$\zeta \triangleq (r_s/r_c)(>1) \quad (5)$$

the law of cosines gives

$$d_c/r_c = (\zeta^2 + 1 - 2\zeta\cos\delta)^{1/2}\ R(\zeta,\delta)^{-1} \quad (6)$$

for a "normalized" slant range. Assuming the fov extends from nadir to horizon yields

$$0 \leq \delta \leq \Delta = \cos^{-1} 1/\zeta \quad (7)$$

From Eq. (1), we write

$$d_c \cos\alpha = (\boldsymbol{r}_c - \boldsymbol{r}_s) \cdot \hat{\boldsymbol{x}}_\odot = r_c \cos\gamma - r_s \cos\nu \quad (8)$$

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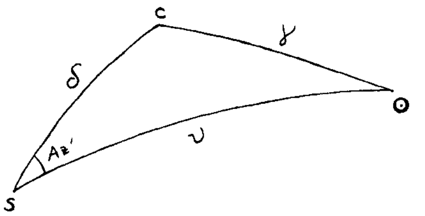

Fig. 2 Spherical triangle with vertices at the Satellite, Cloud, and Sun.

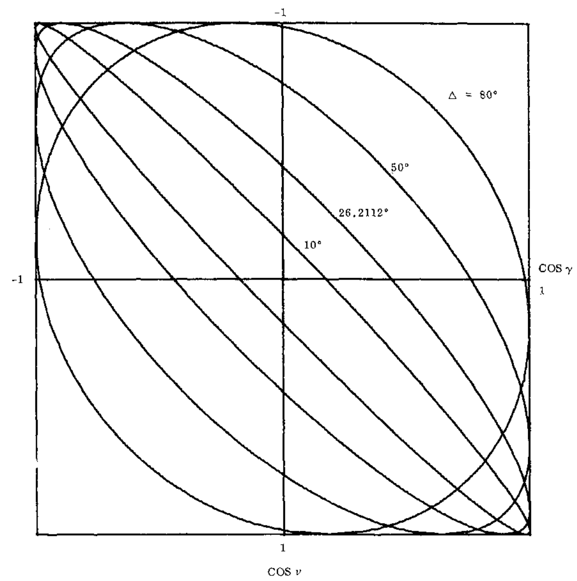

Fig. 3 Region of allowable ν and γ for specified $\Delta(\zeta)$.

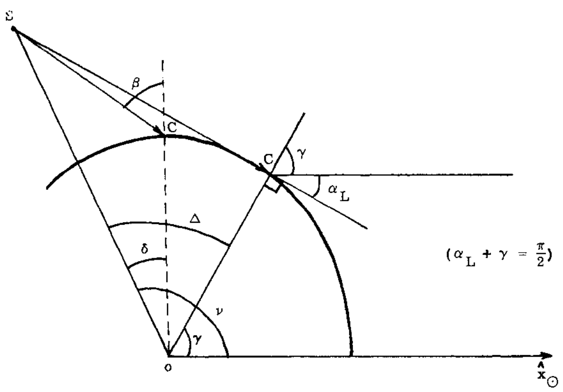

Fig. 4 Viewing geometry for α_L.

so, using Eqs. (5) and (6), α is given by

$$\cos\alpha = R(\zeta,\delta)[\cos\gamma - \zeta\cos\nu] \tag{9}$$

This is a desirable form for α since the effect on α of ν, γ, δ, and ζ can be easily assessed. It is interesting that cos α is simply a linear combination of cos ν and cos γ. Requiring the cloud to be illuminated means $0 \le \gamma \le \pi/2$ so we have a three-dimensional rectangle[4,5] bounded by $-1 \le \cos\nu \le 1$, $0 \le \cos\gamma \le 1$, and $-1 \le \cos\alpha \le 1$.

The earlier lower bound for α was obtained as follows. Since cos α is to be maximized, replace cos γ by one and maximize $R(\zeta,\delta)$; from Eq. (6) this corresponds to the minimum d_c which obviously occurs for $\delta = 0$. Hence

$$\cos\alpha \le \cos\alpha_L = (1 - \zeta\cos\nu)/(\zeta - 1) \tag{10}$$

For a low-altitude satellite, however, ζ is only slightly larger than one and Eq. (10) can be used only for very small ν.

Referring to Fig.2, we see that $0 \le Az' \le \pi$ gives

$$\nu - \delta \le \gamma \le \nu + \delta \tag{11}$$

which are extended, using Eq. (7), to

$$\nu - \Delta \le \gamma \le \nu + \Delta \tag{12}$$

so Eq. (12) imposes restrictions on the available parameter space cos ν, cos γ, cos α. Then

$$\cos(\nu + \Delta) \le \cos\gamma \le \cos(\nu - \Delta) \tag{13}$$

so, for a given $\Delta(\zeta)$, the two-dimensional parameter space cos ν, cos γ is bounded by the curves shown in Fig.3. The allowable region is inside each oval shape† The limiting values $\Delta = 0(\zeta = 1)$ and $\Delta = \pi/2(\zeta = \infty)$ give respectively $\cos\gamma = \cos\nu$, a diagonal straight line and $-\sin\nu \le \cos\gamma \le \sin\nu$ or $\cos^2\nu + \cos^2\gamma = 1$, a circle of radius one centered at the origin.

In order to minimize α, Fig.2 tells us that $Az' = 0$ and $\gamma = \nu - \delta$. Thus, we have a planar configuration shown in Fig. 4 which shows that, for a given ν, γ should be minimized and ν should be maximized. These observations also agree with Eq. (9). The minimum γ occurs for $\delta = \Delta$ so the lower bound is

$$\cos\alpha \le \cos\alpha_L = [\cos(\nu - \Delta) - \zeta\cos\nu]/(\zeta^2 - 1)^{1/2} \tag{14}$$

It is interesting that the dependence on γ is more important than the dependence on δ in Eq. (9), especially for satellites in low orbit where ζ is near one. The inequalities

$$1/(\zeta^2 - 1)^{1/2} = R(\zeta,\Delta) \le R(\zeta,\delta) \le R(\zeta,0) = 1/(\zeta - 1) \tag{15}$$

are obtained from Eq. (6) but, from Eq. (14), it is seen that the *smaller* value of R is used. Also, α takes its minimum value on the right boundary in Fig. 3. Using

$$\cos\Delta = c_\Delta = 1/\zeta; \quad \sin\Delta = s_\Delta = [1 - 1/\zeta^2]^{1/2} \tag{16}$$

in Eq. (14) gives

$$\begin{aligned} \cos\alpha_L &= [(1/\zeta)c_\nu + (1/\zeta)s_\nu(\zeta^2 - 1)^{1/2} - \zeta c_\nu] \\ &= (1/\zeta)[-c_\nu(\zeta^2 - 1)^{1/2} + s_\nu] \\ &= -c_\nu s_\Delta + s_\nu c_\Delta = \cos[\pi/2 - (\nu - \Delta)] \end{aligned} \tag{17}$$

so

$$\alpha_L = \pi/2 - \nu + \Delta = \pi/2 - \gamma \tag{18}$$

in agreement with Fig. 4.

In conclusion, this analysis shows that minimum α is obtained on the satellite horizon with δ a maximum and, furthermore, $\alpha = 0$ is only achieved as the satellite terminator crosses the Sun terminator.

References

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[4] Hitzl, D.L., "The Geometry of Solar Reflections for an Orbiting Sensor," *Journal of the Astronautical Sciences,* Sept.-Oct. 1973, Vol. 21, pp. 65-98.

[5] Hitzl, D.L., "Bounds for the Solar Scatter Angle Observed from Earth Orbit," AIAA Paper 74-843, Anaheim, Calif. 1974.

†Modified slightly in Ref. 5. Note also: $\Delta = 26.2112°$ corresponds to a 400 nm alt orbit.